MATHEMATICAL MODEL FOR THE INTELLIGENT TRAFFIC MANAGEMENT SYSTEM OF THE BUSES

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Abstract
This paper is devoted to improving the mathematical support for the intelligent traffic management system of city buses. It provides an overview of what is the real state of the-art with respect to traffic flow theory. A new mathematical model of the buses motion has been generating in consideration of stochastic factors. The model allows the calculating immediate changes in the city buses schedules connected with speed parameters. The priority will be giving to buses at traffic lights helping them to run on schedule, because traffic lights are able to adapt to the traffic flow and maintain its optimal levels. The check of mathematical model’s adequacy is proposing on the example of the Lugansk region cities (Ukraine). The algorithm and computer program have been developing, that realize the modeling module using a programming environment Delphi. The model and program realization make allowance for increasing the efficiency of passenger service when projecting city passenger transports. With regard to the traffic organization, the automated control system as the element of the intelligent transport systems plays the increasingly important role as a key component of the transport system, which is able to form the right choice for customers across a network, to support safe travel.

Keywords: modeling, intelligent transport systems, information systems, dispatching control, transport flow, passenger flow

1 INTRODUCTION
Every system of management in terms of technology of its operation resolves three main tasks: collection and transmission of information about the controlled object, processing of information and, finally, issue of control actions on the object of control. Automated control system (ACS) automates all the steps. This is the first to differ ACS from the simple use of computers in management. The simple using of computers is to solve individual problems of management, i.e., processing of information, but stages of gathering information and formation of control actions are not automated here.

There are two basic types of control systems: technological process control systems (TPCS) and organizational or administrative control systems (ACS) (Smulders, 1989).
Recently, there is a trend to merger ITMS and ACS into a single integrated management system, see fig. 1.

Fig. 1 Intelligent traffic management system of the buses

During the last years developers increasingly have to solve problems of designing control system of so complex nature that it became simply impossible to use for design purposes old experienced methods (Nikolaev, Aleksakhin, Kuznetsov, & Stroganov, 2003).

Deterministic mathematical programming models are often inadequate to real technological processes. This is due, primarily, to the inaccuracy of the probabilistic nature of the indications and limitations that are imposed on the model of the enterprise. Planning based on the most optimistic forecasts, the factors contributing to the production and consumption, often turns out to be untenable because of the lack of provision for the correction of inconsistencies that arise when any link in the chain of production yields less than expected for the optimistic forecast. It is much better to plan for the average performance if their spread is large enough. And in this case the real values of the indicators may differ significantly from the mean values, and the proposed plan would be unworkable (Closs, Davidson, Dawe, Templeton, & Levitt, 2007).

2 MATHEMATICAL MODELING OF DRIVING TRAFFIC FLOWS

Every system of control in terms of its functioning technology solves three main tasks: the collection and transmission of information about the controlled object, information processing, and finally the issue of control actions on the object of control. Automated Control System (ACS) automates all these steps (Mikulski, 2011).

There are two basic types of automated control systems: technological process control systems (TPCS) and organizational or administration

In recent years the trend to merger control system into a single ACS integrated control systems. With such a confluence of all the majority of circulating in the system information is transmitted in the form of signals, and special types of documents on computer media. Thus, the boundary between TPCS and ACS to a certain extent is blurred.

The inclusion of computers in an object control system requires the developer to do a lot of effort. First, to fill it with programs related to technology management, align it with the dynamics of the dynamic characteristics of a real object, and finally to agree on the physical form of the signals coming from the object into the computer and issued from the computer to an object (Transportation & Logistics 2030 Volume 4: Securing the supply chain, 2012, pp. 25-29) (OECD, 2002).

2.1 Review and analysis of previous studies and statement of the problem

If the control algorithm is stringent, quite simple and fully known to the customer and the developer, it is very easy to program a computer work, laying in her memory, for example, the table given automaton encoding of inputs and outputs associated with the object of control (Smulders, 1989). Coding of internal states, there may be arbitrary.

If the control algorithm is not stringent, if in the process of decision to grant the object of control signals to solve various optimization problems, if, finally, in the first stage of experimental verification of the control system is an accumulation of new information about an object, previously unknown to the customer or developer, then the use of computers is warranted. Suppose that at some discrete moment of time \( t \) (since the computer works in discrete time, it makes sense to consider the work of the entire control system in discrete time) to the input of the control system receives a set of signals, characterized by the vector: 

\[
\{a_1, \ldots, a_n, \beta_1, \ldots, \beta_m, r_1, \ldots, r_q\}
\]

We will call this set the situation at the moment \( t \) and write it as \( S(t) \) (Mohan, Padmanabhan, & Ramjee, 2008).

As an instant photograph \( S(t) \) contains all the information about the object of control, which can be collected at this time. The task of management is that on the basis of knowledge about \( S(t) \), and some additional knowledge about the object to give the smallest possible delay of the control actions on the object, forming a vector \( \{u_1, u_2, \ldots, u_j\} \). This vector is called a solution at the moment \( t \) and will be denoted as \( U(t) \).

Formally, the control problem is that when a computer at the entrance of the situation \( S(t) \) gave out at the output of the computer some solution \( U(t) \) that would be technically acceptable, if there are feasible solutions, gave out the best solution in terms of some criterion of management. If you are given a choice of \( U(t) \) to \( S(t) \) single-valued and the process of selecting a developer known for, it remains only to program this process and to put it in the computer memory (Al-Khateeb, Johari, & Al-Khateeb, 2008).

The described approach requires that to the computer memory was added a table on the left side that lists all the valid \( S(t) \) function object technology, and on the right - are the sets of solutions that are technologically acceptable for a given situation, and of which then need to select the best from the point of view control criterion F (Tyagi, Kalyanaraman, & Krishnapuram, 2012). But, unfortunately, for any real control object the size of this table will be astronomically high. We calculate, for example, the number of possible situations that may arise at the intersection. Apparently, for most real-world objects for which you want to build a control system, we have the inequality \( |S(t)| \geq |U(t)| \).

Thus, the approach associated with the introduction of computer memory decision table, is ineffective. It is therefore necessary to look for a different approach to solving the problem (Tarnoff, Bullock, Young, & al., 2009). This other approach involves the introduction of computer memory of the two models: the object model, control and management decisions. Object model in the computer memory during calculation of the output values of the results obtained from the model of decision-making, should mimic the real behavior of the object, predicting the output values of the
object, resulting from exposure to the object of the control system. These predicted values of the outputs of the object, forming a vector \( \{ \beta_1(t + 1), \beta_2(t + 1), \ldots, \beta_m(t + 1) \} \) fed to the decision-making model and also on the block correction. Correction unit compares the prediction derived from the model of the object with the real values of the output object, which will result in some control actions on the object of control (Joshi, Rajamani, Takayuki, Prathapaneni, & Subramaniam, 2013). Information about the mismatch of prediction and reality is transferred to the model of decision making. This information forms a vector \( \{ w_1, w_2, \ldots, w_h \} \). If necessary, decision-making model provides an adjustment of the object model.

### 2.2 Development of mathematical models used in ACS for transport

The flow can be considered to consist of any number of types of cars. In this model it is assumed that the entire stream of cars consists of three groups of speed, relevant passenger, medium and heavy freight trucks. The distribution of intervals out of cars on the road can be accepted under any law. In the model there are the following notations (Nikolaev, Aleksakhin, Kuznetsov, & Stroganov, 2003):

- \( N_1 \) – traffic density in the direction AB; \( N_2 \) – traffic density in the direction of the BA; \( T \) – the duration of monitoring of movement of the flow of cars; \( m \) – the number of required calculations on the model; \( M \) – the number of counted realizations at a given moment of time; \( T_1 \) – the moment of transition of the entire system, which simulates the flow from one state to another; \( \Delta t_1 \) – a random time interval between successive moments of departure of cars from point A; \( \Delta t_2 \) – the same from point B; \( \tau_1 \) – minimal time spent in the current state of the vehicle for guiding AB (e.g., time spent in a pack); \( \tau_2 \) – the same for direction BA; \( n_1 \) – number of cars (buses) of the first group, traveling the path AB for \( m \) implementations in both directions; \( n_2 \) – the same for the second cars speed group; \( n_3 \) – the same for third cars speed group \( t_1 \) – the total time of movement of all \( n_1 \) cars of the first group \( t_2 \) – the same for all \( n_2 \) vehicles of the second group \( t_3 \) – the same for all \( n_3 \) vehicles of the third group.

The free speed of cars in each group are taken to depend only on the geometric elements of the road and the availability of funds and the organization of the movement can be determined by any method or calculated according to the experimental observations made in real-world driving conditions, taking into account the impact of signs and markings.

In modeling flow of cars is determined by the state at the time \( T_1 \). In this case the ground state of flow of cars characterized by the number of cars on the road under consideration and the condition of each vehicle (moving in free conditions or in confined spaces.) For each vehicle and each of its states is calculated duration of its stay in this state.

Thus, if at any moment of time the flow of cars was in a ground state, then move it to another ground state requires that one of the cars moved to a new state (for example, overtook a column cars), or on the road drove a new car from the end points.

In view of the above simulation is performed as follows:

determine the coordinates of the geometric elements of the analyzed road and siting of traffic signs and markings in the light zone of their action;

depending on the intensity of motion take the distribution of intervals, that the cars come through on the road from the end-points;

using the model described above analyze and calculate the basic characteristics of the flow of vehicles;

receive results of the models that describe the conditions of the flow of cars on the road under consideration.

As a result, simulation can be obtained from the following data characterizing the conditions of the flow of cars on the road: the average duration of each type of car traffic, loss of time each type of vehicle as a result of movement of these vehicles in the stream, with an average speed of flow of vehicles and every type of cars.
Consider the relative frequency of zero and exponentially distributed intervals between successive cars. Since the ratio of the number of fast cars to the number of slow cars equal to $p$, then the probability that a randomly selected car is fast equal to $p/(1 + p)$. Consequently, the probability interval is zero equal to $p/(1 + p)$, and the probability of an exponentially distributed interval equal to $1/(1 + p)$, and we can write the density distribution of intervals between successive cars both types

$$
g(x) = \frac{p}{1 + p} \delta(x) + \frac{\lambda e^{-\lambda x}}{1 + p}. \tag{1}$$

Distribution of the number of cars can be found using the method of differential-difference equations, denoting $p_n(\tau)$ the probability that the interval of time $\tau$ will be regarded $n$ automobiles. Since the probability of a lack of cars during the period of time $\Delta\tau$ equal to

$$(1 - \lambda)\Delta\tau + o(\Delta\tau), \tag{2}$$

the likelihood of a vehicle during the time interval $\Delta\tau$ equal to

$$(1 - \lambda)\lambda \Delta\tau + o(\Delta\tau), \tag{3}$$

the likelihood of two vehicles for a time $\Delta\tau$ equal to

$$p\Delta\tau + o(\Delta\tau), \tag{4}$$

probability of more than two cars during the time interval $\Delta\tau$ equal to

$$o(\Delta\tau), \tag{5}$$

you can easily get the equation

$$\begin{align*}
\frac{\partial p_n(\tau)}{\partial \tau} &= -\lambda p_n(\tau) + \lambda (1 - p)p_{n-1}(\tau) + \\
&\quad + p\lambda p_{n-2}(\tau) \\
&= n = 0, 1, \ldots, \tag{6}
\end{align*}$$

where $p_n(\tau) = 0$, if $n < 0$. Solving the system of equations (2-6) by successive substitutions, we obtain

$$p_n(\tau) = e^{-\lambda\tau} \sum_{i=0}^{N} \frac{p_i}{(n-i)!(2i-n)!} (\lambda\tau)^i, \tag{7}$$

where $N$ – integral part of $\frac{1}{2}(n + 1)$.

We assume that all vehicles traveling in a northerly direction, have the same speed $U$, except the only controlling car moving in a northerly direction and having a free running speed $V > U$. Assume further that these cars have the same arrangement as in the queuing model with an independent $(D, M)$ flow.

Queuing model with an independent traffic flow is constructed as follows: each interval is chosen by performing a Bernoulli experiment with the probabilities of outcomes $p$ and $q$. If the experience a successful outcome, we selected the interval with density $g_1$, in the case of a bad experience with the outcome of the selected interval of the density distribution $g_2$. Cars, the density distribution of the intervals for which equal to $g_1$ regarded as forming one and the same place, and cars with the density distribution of the intervals $g_2$ are considered as forming a separate queue.

The most important case of the queuing model of an independent traffic flow is such that when the density function $g_2$ is deterministic, and function $g_2$ – purely random laws of $(D - process$ and deterministic $\Delta - stochastic process)$. If $M$ – the length of the car, and $\lambda$ – the parameter of the Poisson distribution, the density distribution of intervals between successive cars of both types has the form

$$g(x) = p\delta(x - \Delta) + q\lambda e^{-\lambda(x - \Delta)}, x \geq \Delta. \tag{8}$$

This model will be called the queuing system with an independent $(D, M)$ traffic flow.

$\lambda$ – flux density, $\rho$ – intensity, and $\Delta$ – minimum distance between consecutive cars. The length of the queue has a geometric distribution, and therefore, the average queue length in terms of the number of vehicles is equal to

$$\mu = \frac{1}{1 - \lambda\Delta}, \tag{9}$$

by distance

$$\frac{\Delta}{1 - \lambda\Delta}, \tag{10}$$

by time

$$\frac{\Delta}{U} \frac{1}{1 - \lambda\Delta}. \tag{11}$$
In addition, the average distance between the queues is equal to \( \frac{1}{\lambda} \), that is, the average distance between consecutive cars. The distance between successive queues of different vehicles is not the distance between the queues; the first buffer is measured between the front car, and the second - from the front buffer to the next car back buffer. Therefore, they differ in magnitude \( \Delta \).

In order the car control which a speed boost, equal to \( V - U \), will be able to beat the queue and to overcome the distance between the bursts, it will take time

\[
\frac{\Delta}{1 - \lambda \Delta} + \frac{1}{V - U} .
\]  

(12)

Thus ends the part of a typical cycle, when the control car has a free running speed. If \( W \) – the average waiting time (the time during which the rate is equal to \( U \)), and \( m \) – achieved average speed, we obtain

\[
m = \frac{\lambda U W (V - U)(1 - \lambda \Delta) + V}{V W (V - U)(1 - \lambda \Delta) + 1} .
\]  

(13)

This expression is equal to \( V \) in the following special cases: \( U = V \) (the car does not make overtaking) and \( \lambda \Delta = 1 \) (traffic jam).

In this case, the problem of finding \( m \) reduces to the problem of finding a definition \( W \). However, before you perform these calculations, we put in the formula (13) values \( W \) obtained for the hypothesis of Carlson (repetition time, is equal to \( k V X (V - X) \)) and the hypothesis of equal delay (travel time is equal to \( k V \)). Since it is assumed that these expressions apply in the case of a simple overtaking, then before you make a substitution in the formula (13), multiply them by the average queue length.

For hypothesis of Carlson (Kerner & Rehborn, 1996) we have

\[
W = \frac{k V}{U} \frac{1}{V - U} \frac{1}{1 - \lambda \Delta} .
\]  

(14)

where \( k \) – constant, meaning of which is not grasped intuitively, but it has the same dimension as \( \Delta \left( \frac{\text{distance}}{\text{number of cars(buses)}} \right) \).

The corresponding value \( m \) is equal to

\[
m = UV \frac{1 + k \lambda}{U + k \lambda V} .
\]  

(15)

Formula (15) gives the results that make sense in certain limiting cases: if \( V \rightarrow U \), then \( m \rightarrow V \), if \( V \rightarrow \infty \) then

\[
m_{\text{max}} = U \frac{\lambda k + 1}{\lambda k} .
\]  

(16)

However, if we stop the flow of North, setting \( U = 0 \), we find that the test car also stops, we get the result, which is absolutely not true. The reason for this situation is quite simple: these drawbacks inherent in the hypothesis of Carlson, and the expression (16) are independent of the southern stream. Consequently, it is unreasonable to expect that if \( U \) (and also \( \lambda \)) change, we obtain results that will make sense. Note that in fact when \( \lambda = 0 \) we get the correct value, and if \( \lambda = \lambda_{\text{max}} = 1/\Delta \) the correct value is not get.

The hypothesis of equal delay gives the expression

\[
m = V \frac{1 + k \lambda U}{1 + k \lambda V} .
\]  

(17)

This result is very similar to the formula (17). In this case \( k \) has the same dimension as \( W \), and when \( U = 0 \) we get a completely consistent value for \( m \).

Let us find the average length of delay due to a simple pass. The same model with the parameters of the queuing \( \lambda' \) and \( U' \) will be used for the southern stream. It is easy to show that such a structure for controlling a vehicle provided with the time intervals, the density distribution which has the form

\[
g(x) = p \delta(x - \sigma) + q \beta e^{-\beta(x - \sigma)}
\]  

(18)

with an appropriate choice of values \( p \), \( q \), \( \sigma \) and \( \beta \). Since the flow of the southern approaches to the control vehicle with speed \( V + U' \) and the average queue length of the southern stream of
cars is equal to \((1 - \lambda')\Delta^{-1}\), respectively, the probability of large and small intervals between successive vehicles are
\[
p = \lambda' \Delta \quad \text{and} \quad q = 1 - \lambda' \Delta,
\]
average length of small intervals
\[
\sigma = \frac{\Delta}{V + U'}, \quad (19)
\]
and the reciprocal of the mean length of big spacing,
\[
\beta = \lambda'(V + U'). \quad (20)
\]
Assuming the car to overtake, to perform the maneuver must have a distance equal to the length of the completed vehicle, we can eliminate the unknown constant and find the time required for passing:
\[
T = \frac{\Delta}{V - U} > \sigma. \quad (21)
\]
Distribution density given by (21) corresponds to the distribution function
\[
G(x) = \begin{cases} 
1 - q e^{-\beta(x - \sigma)}, & x > \sigma, \\
0, & x < \sigma.
\end{cases} \quad (22)
\]
Mathematical expectation is
\[
p \sigma + \frac{q}{\beta} (1 - \beta q).
\]
The corresponding density function of the initial interval has the form
\[
g_0(x) = \begin{cases} 
\frac{q \beta}{q + \sigma \beta} e^{-\beta(x - \sigma)}, & x > \sigma, \\
\frac{\beta}{q + \sigma \beta}, & x < \sigma.
\end{cases} \quad (23)
\]
The distribution function of the initial interval has the form
\[
G_0(x) = \begin{cases} 
1 - \frac{q \beta}{q + \sigma \beta} e^{-\beta(x - \sigma)}, & x > \sigma, \\
\frac{\beta x}{q + \sigma \beta}, & x < \sigma.
\end{cases} \quad (24)
\]
We find that in the case of a simple overtaking the average delay time is
\[
\frac{\sigma^2 \beta}{2} = \frac{1 + \beta T + p \beta \sigma}{\beta} + \frac{\exp \beta (T - \sigma)}{q \beta}. \quad (25)
\]
We substitute the value of a new permanent, we obtain
\[
W = \frac{\exp\left[\Delta \lambda' \frac{U + U'}{V - U}\right]}{(1 - \lambda') (1 - \lambda) \lambda' (V + U')} - \frac{(\Delta \lambda')^2 (V - U) + 2 (V - U + \Delta \lambda V + \Delta \lambda' U')}{(1 - \lambda \Delta) (2 \lambda') (V + U') (V - U)}. \quad (26)
\]
We turn now to the more general case, which is characterized by a density of distribution of the velocity of free motion \(f_0(x)\). In this case it is impossible to obtain an expression for \(W\). The flux density of vehicles, following a rate \(X\) equal to \(\rho/m(X)\).

The car, having a free running speed \(V\), overtaking these cars with the average frequency \(\rho(V, X)\) \(\rho(V, X)\) – proportion of time during which the car has a free running speed \(V\), is the head) can be expressed in terms of function \(\rho(V, X)\) by the relation
\[
1 - \alpha(V) = \int_0^V \rho(V, x) W f_0(x) dx, \quad (28)
\]
where \(W\) – function of average waiting time (the time during which the rate is \(U\)), it has one or more arguments, depending on where we are forced to make an assumption about the time sequence. Similarly, the expression for the quantity \(s(V)\) \(s(V)\) – the average distance traveled per unit of time driving, having a free running speed, as a slave) can be written as
\[
s(V) = \int_0^V \rho(V, x) x W f_0(x) dx. \quad (29)
\]
Considering formulas (15) - (17) together with the formula
\[
m(V) = V \alpha(V) + s(V),
\]
we obtain an integral equation for \(m(V)\):
In the DELPHI programming environment we created a complex of problem-oriented software for modeling of intelligent traffic management system (ITMS).

3 IMPLEMENTATION OF THE MODEL

Program system “Automatic bus Controller systems” is developed in visual design environment Delphi, fig. 2. Object-oriented programming and visual design allow creating excellent applications for different purposes (Lakhno & Pilipenko, 2007).

The main features of software owned by the possibility of calculating the operational control tower operator changes the timetable of mobile units to reflect the changes of the velocity parameters of each of the high-speed units.

The system “Automatic bus Controller systems”, designed for client-server platform. So before you start working with the program, you must specify the server address on which the system is installed. Later in this guide assumes that the system is installed at localhost, which means on the computer on which you are directly working with. The information system „Automatic bus Controller systems“ used during the period from 2010 to 2012.

Implementation of the results of research, greatly improving the quality of transport services in the cities of Lugansk region. The economic impact exceeded $ 20,000 per month, labor productivity increased by 12.1%.

The results obtained in the studies are shown in Fig. 3.

![Fig. 2 Program system “Automatic bus Controller systems” (Lahno V., Pilipenko A.,2011)](image)

![Fig. 3 The results of the study](image)

The system “Automatic bus Controller systems” consists of separate windows, each of which is intended to carry out its functional tasks - viewing and editing the contents of the database, calculate the interval of motion of moving units, etc.

4 CONCLUSIONS

During the last years developers of automated information systems in transport is increasingly have to solve problems of designing control
system of so complex nature that it became simply impossible to use for design purposes old experienced methods.

The article describes some of the features of modeling of traffic flows using the stochastic simulation. The basic computational function, and describes a computer program for managing passenger transport, which is implemented, this mathematical model.

5 WORKS CITED


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